

# Bürgi's sexagesimal multiplication table in his *Fundamentum Astronomiæ*

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13 December 2015

Bürgi's *Fundamentum Astronomiæ* manuscript is kept at the Biblioteka Uniwersytecka Wrocław, under call number IV Qu 38a. The sexagesimal multiplication table given on pages 9v-15r was provided by Dieter Launert and is included in LOCOMAT (<http://locomat.loria.fr>) with permission.

For more on these tables, see Menso Folkerts, Dieter Launert, and Andreas Thom, "Jost Bürgi's Method for Calculating Sines", *Historia Mathematica* (to appear, a preprint is available on arXiv under id 1510.03180v1).

Bürgi's entire manuscript is available at

<http://www.bibliotekacyfrowa.pl/dlibra/doccontent?id=45624>

| 1                        | 2                           | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------------------------|-----------------------------|---|---|---|---|---|---|---|----|
| 0 1 0 2 0 3 0 4 0 5      | 1 0 6 0 7 0 8 0 9 0 10      |   |   |   |   |   |   |   |    |
| 0 2 0 4 0 6 0 8 0 10     | 2 0 12 0 14 0 16 0 18 0 20  |   |   |   |   |   |   |   |    |
| 0 3 0 6 0 9 0 12 0 15    | 3 0 18 0 21 0 24 0 27 0 30  |   |   |   |   |   |   |   |    |
| 0 4 0 8 0 12 0 16 0 20   | 4 0 24 0 28 0 32 0 36 0 40  |   |   |   |   |   |   |   |    |
| 0 5 0 10 0 15 0 20 0 25  | 5 0 30 0 35 0 40 0 45 0 50  |   |   |   |   |   |   |   |    |
| 0 6 0 12 0 18 0 24 0 30  | 6 0 36 0 42 0 48 0 54 1 0   |   |   |   |   |   |   |   |    |
| 0 7 0 14 0 21 0 28 0 35  | 7 0 42 0 49 0 56 1 3 1 10   |   |   |   |   |   |   |   |    |
| 0 8 0 16 0 24 0 32 0 40  | 8 0 48 0 56 0 4 1 12 1 20   |   |   |   |   |   |   |   |    |
| 0 9 0 18 0 27 0 36 0 45  | 9 0 54 0 1 3 1 12 1 21 1 30 |   |   |   |   |   |   |   |    |
| 0 10 0 20 0 30 0 40 0 50 | 10 1 0 1 10 1 20 1 30 1 40  |   |   |   |   |   |   |   |    |
| 0 11 0 22 0 33 0 44 0 55 | 11 1 6 1 17 1 28 1 39 1 50  |   |   |   |   |   |   |   |    |
| 0 12 0 24 0 36 0 48 1 0  | 12 1 12 1 24 1 36 1 48 2 0  |   |   |   |   |   |   |   |    |
| 0 13 0 26 0 39 0 52 1 5  | 13 1 18 1 31 1 44 1 57 2 10 |   |   |   |   |   |   |   |    |
| 0 14 0 28 0 42 0 56 1 10 | 14 1 24 1 38 1 52 2 6 2 20  |   |   |   |   |   |   |   |    |
| 0 15 0 30 0 45 1 0 1 15  | 15 1 30 1 45 2 0 2 15 2 30  |   |   |   |   |   |   |   |    |
| 0 16 0 32 0 48 1 4 1 20  | 16 1 36 1 52 2 8 2 24 2 40  |   |   |   |   |   |   |   |    |
| 0 17 0 34 0 51 1 8 1 25  | 17 1 42 1 59 2 16 2 33 2 50 |   |   |   |   |   |   |   |    |
| 0 18 0 36 0 54 1 12 1 30 | 18 1 48 2 6 2 24 2 42 3 0   |   |   |   |   |   |   |   |    |
| 0 19 0 38 0 57 1 16 1 35 | 19 1 54 2 13 2 32 2 51 3 10 |   |   |   |   |   |   |   |    |
| 0 20 0 40 1 0 1 20 1 40  | 20 2 0 2 20 2 40 3 0 3 20   |   |   |   |   |   |   |   |    |
| 0 21 0 42 1 3 1 24 1 45  | 21 2 6 2 27 2 48 3 9 3 30   |   |   |   |   |   |   |   |    |
| 0 22 0 44 1 6 1 28 1 50  | 22 2 12 2 34 2 56 3 18 3 40 |   |   |   |   |   |   |   |    |
| 0 23 0 46 1 9 1 32 1 55  | 23 2 18 2 41 3 4 3 27 3 50  |   |   |   |   |   |   |   |    |
| 0 24 0 48 1 12 1 36 2 0  | 24 2 24 2 48 3 12 3 36 4 0  |   |   |   |   |   |   |   |    |
| 0 25 0 50 1 15 1 40 2 5  | 25 2 30 2 55 3 20 3 45 4 10 |   |   |   |   |   |   |   |    |
| 0 26 0 52 1 18 1 44 2 10 | 26 2 36 3 2 3 28 3 54 4 20  |   |   |   |   |   |   |   |    |
| 0 27 0 54 1 21 1 48 2 15 | 27 2 42 3 9 3 36 4 3 4 30   |   |   |   |   |   |   |   |    |
| 0 28 0 56 1 24 1 52 2 20 | 28 2 48 3 16 3 44 4 12 4 40 |   |   |   |   |   |   |   |    |
| 0 29 0 58 1 27 1 56 2 25 | 29 2 54 3 23 3 52 4 21 5 50 |   |   |   |   |   |   |   |    |
| 0 30 1 0 1 30 2 0 2 30   | 30 3 0 3 30 4 0 4 30 5 0    |   |   |   |   |   |   |   |    |

| 1                        | 2                           | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------------------------|-----------------------------|---|---|---|---|---|---|---|----|
| 0 31 1 2 1 33 2 4 20 35  | 31 3 6 3 37 4 8 4 39 5 10   |   |   |   |   |   |   |   |    |
| 0 32 1 4 1 36 2 8 2 40   | 32 3 12 3 44 4 16 4 48 5 20 |   |   |   |   |   |   |   |    |
| 0 33 1 6 1 39 2 12 2 45  | 33 3 18 3 51 4 24 4 57 5 30 |   |   |   |   |   |   |   |    |
| 0 34 1 8 1 42 2 16 2 50  | 34 3 24 3 58 4 32 5 6 5 40  |   |   |   |   |   |   |   |    |
| 0 35 1 10 1 45 2 20 2 55 | 35 3 30 4 5 4 40 5 15 5 50  |   |   |   |   |   |   |   |    |
| 0 36 1 12 1 48 2 24 3 0  | 36 3 36 4 12 4 48 5 24 6 0  |   |   |   |   |   |   |   |    |
| 0 37 1 14 1 51 2 28 3 5  | 37 3 42 4 19 4 56 5 33 6 10 |   |   |   |   |   |   |   |    |
| 0 38 1 16 1 54 2 32 3 10 | 38 3 48 4 26 5 4 5 42 6 20  |   |   |   |   |   |   |   |    |
| 0 39 1 18 1 57 2 36 3 15 | 39 3 54 4 33 5 12 5 51 6 30 |   |   |   |   |   |   |   |    |
| 0 40 1 20 2 0 2 40 3 20  | 40 4 0 4 40 5 20 6 0 6 40   |   |   |   |   |   |   |   |    |
| 0 41 1 22 2 3 2 44 3 25  | 41 4 6 4 47 5 28 6 9 6 50   |   |   |   |   |   |   |   |    |
| 0 42 1 24 2 6 2 48 3 30  | 42 4 12 4 54 5 36 6 18 7 0  |   |   |   |   |   |   |   |    |
| 0 43 1 26 2 9 2 52 3 35  | 43 4 18 5 1 5 44 6 27 7 10  |   |   |   |   |   |   |   |    |
| 0 44 1 28 2 12 2 56 3 40 | 44 4 24 5 8 5 52 6 36 7 20  |   |   |   |   |   |   |   |    |
| 0 45 1 30 2 15 3 0 2 45  | 45 4 30 5 15 6 0 6 45 7 30  |   |   |   |   |   |   |   |    |
| 0 46 1 32 2 18 3 4 3 50  | 46 4 36 5 22 6 8 6 54 7 40  |   |   |   |   |   |   |   |    |
| 0 47 1 34 2 21 3 8 3 55  | 47 4 42 5 29 6 16 7 3 7 50  |   |   |   |   |   |   |   |    |
| 0 48 1 36 2 24 3 12 4 0  | 48 4 48 5 36 6 24 7 12 8 0  |   |   |   |   |   |   |   |    |
| 0 49 1 38 2 27 3 16 4 5  | 49 4 54 5 43 6 32 7 21 8 10 |   |   |   |   |   |   |   |    |
| 0 50 1 40 2 30 3 20 4 10 | 50 5 0 5 50 6 40 7 30 8 20  |   |   |   |   |   |   |   |    |
| 0 51 1 42 2 33 3 24 4 15 | 51 5 6 5 57 6 48 7 39 8 30  |   |   |   |   |   |   |   |    |
| 0 52 1 44 2 36 3 28 4 20 | 52 5 12 6 4 6 56 7 48 8 40  |   |   |   |   |   |   |   |    |
| 0 53 1 46 2 39 3 32 4 25 | 53 5 18 6 11 7 4 7 57 8 50  |   |   |   |   |   |   |   |    |
| 0 54 1 48 2 42 3 36 4 30 | 54 5 24 6 18 7 12 8 6 9 0   |   |   |   |   |   |   |   |    |
| 0 55 1 50 2 45 3 40 4 35 | 55 5 30 6 25 7 20 8 15 9 10 |   |   |   |   |   |   |   |    |
| 0 56 1 52 2 48 3 44 4 40 | 56 5 36 6 32 7 28 8 24 9 20 |   |   |   |   |   |   |   |    |
| 0 57 1 54 2 51 3 48 4 45 | 57 5 42 6 39 7 36 8 33 9 30 |   |   |   |   |   |   |   |    |
| 0 58 1 56 2 54 3 52 4 50 | 58 5 48 6 46 7 44 8 42 9 40 |   |   |   |   |   |   |   |    |
| 0 59 1 58 2 57 3 56 4 55 | 59 5 54 6 53 7 52 8 51 9 50 |   |   |   |   |   |   |   |    |
| 1 0 2 0 3 0 4 0 5 0      | 60 6 0 7 0 8 0 9 0 10 0     |   |   |   |   |   |   |   |    |

| 11                       | 12                                 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|--------------------------|------------------------------------|----|----|----|----|----|----|----|----|
| 0 11 0 12 0 13 0 14 0 15 | <b>1</b> 0 16 0 17 0 18 0 19 0 20  |    |    |    |    |    |    |    |    |
| 0 22 0 24 0 26 0 28 0 30 | <b>2</b> 0 32 0 34 0 36 0 38 0 40  |    |    |    |    |    |    |    |    |
| 0 33 0 36 0 39 0 42 0 45 | <b>3</b> 0 48 0 51 0 54 0 57 1 0   |    |    |    |    |    |    |    |    |
| 0 44 0 48 0 52 0 56 1 0  | <b>4</b> 1 4 1 8 1 12 1 16 1 20    |    |    |    |    |    |    |    |    |
| 0 55 1 0 1 5 1 10 1 15   | <b>5</b> 1 20 1 25 1 30 1 35 1 40  |    |    |    |    |    |    |    |    |
| 1 6 1 12 1 18 1 24 1 30  | <b>6</b> 1 36 1 42 1 48 1 54 2 0   |    |    |    |    |    |    |    |    |
| 1 17 1 24 1 31 1 38 1 45 | <b>7</b> 1 52 1 59 2 6 2 13 2 20   |    |    |    |    |    |    |    |    |
| 1 28 1 36 1 44 1 52 2 0  | <b>8</b> 2 8 2 16 2 24 2 32 2 40   |    |    |    |    |    |    |    |    |
| 1 39 1 48 1 57 2 6 2 15  | <b>9</b> 2 24 2 33 2 42 2 51 3 0   |    |    |    |    |    |    |    |    |
| 1 50 2 0 2 10 2 20 2 30  | <b>10</b> 2 40 2 50 3 0 3 10 3 20  |    |    |    |    |    |    |    |    |
| 2 1 2 12 2 23 2 34 2 45  | <b>11</b> 2 56 3 7 3 18 3 29 3 40  |    |    |    |    |    |    |    |    |
| 2 12 2 24 2 36 2 48 3 0  | <b>12</b> 3 12 3 24 3 36 3 48 4 0  |    |    |    |    |    |    |    |    |
| 2 23 2 36 2 49 3 2 3 15  | <b>13</b> 3 28 3 41 3 54 4 7 4 20  |    |    |    |    |    |    |    |    |
| 2 34 2 48 3 2 3 16 3 30  | <b>14</b> 3 44 3 58 4 12 4 26 4 40 |    |    |    |    |    |    |    |    |
| 2 45 3 0 3 15 3 30 3 45  | <b>15</b> 4 0 4 15 4 30 4 45 5 0   |    |    |    |    |    |    |    |    |
| 2 56 3 12 3 28 3 44 4 0  | <b>16</b> 4 16 4 32 4 48 5 4 5 20  |    |    |    |    |    |    |    |    |
| 3 7 3 24 3 41 3 58 4 15  | <b>17</b> 4 32 4 49 5 6 5 23 5 40  |    |    |    |    |    |    |    |    |
| 3 18 3 36 3 54 4 12 4 30 | <b>18</b> 4 48 5 6 5 24 5 42 6 0   |    |    |    |    |    |    |    |    |
| 3 29 3 48 4 7 4 26 4 45  | <b>19</b> 5 4 5 23 5 42 6 1 6 20   |    |    |    |    |    |    |    |    |
| 3 40 4 0 4 20 4 40 5 0   | <b>20</b> 5 20 5 40 6 0 6 20 6 40  |    |    |    |    |    |    |    |    |
| 3 51 4 12 4 33 4 54 5 15 | <b>21</b> 5 36 5 57 6 18 6 39 7 0  |    |    |    |    |    |    |    |    |
| 4 2 4 24 4 46 5 8 5 30   | <b>22</b> 5 52 6 14 6 36 6 58 7 20 |    |    |    |    |    |    |    |    |
| 4 13 4 36 4 59 5 22 5 45 | <b>23</b> 6 8 6 31 6 54 7 17 7 40  |    |    |    |    |    |    |    |    |
| 4 24 4 48 5 12 5 36 6 0  | <b>24</b> 6 24 6 48 7 12 7 36 8 0  |    |    |    |    |    |    |    |    |
| 4 35 5 0 5 25 5 50 6 15  | <b>25</b> 6 40 7 5 7 30 7 55 8 20  |    |    |    |    |    |    |    |    |
| 4 46 5 12 5 38 6 4 6 30  | <b>26</b> 6 56 7 22 7 48 8 14 8 40 |    |    |    |    |    |    |    |    |
| 4 57 5 24 5 51 6 18 6 45 | <b>27</b> 7 12 7 39 8 6 8 33 9 0   |    |    |    |    |    |    |    |    |
| 5 8 5 36 6 4 6 32 7 0    | <b>28</b> 7 28 7 56 8 24 8 52 9 20 |    |    |    |    |    |    |    |    |
| 5 19 5 48 6 17 6 46 7 15 | <b>29</b> 7 44 8 13 8 42 9 11 9 40 |    |    |    |    |    |    |    |    |
| 5 30 6 0 6 30 7 0 7 30   | <b>30</b> 8 0 8 30 9 0 9 30 10 0   |    |    |    |    |    |    |    |    |

| 11                            | 12                                      | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|-------------------------------|---|----|----|----|----|----|----|----|----|
| 5 41 6 17 6 43 7 14 7 45      | <b>31</b> 8 16 8 47 9 18 9 49 10 20     |    |    |    |    |    |    |    |    |
| 6 52 6 24 6 56 7 28 8 0       | <b>32</b> 8 32 9 4 9 36 10 8 10 40      |    |    |    |    |    |    |    |    |
| 6 3 6 36 7 9 7 42 8 15        | <b>33</b> 8 48 9 21 9 54 10 27 11 0     |    |    |    |    |    |    |    |    |
| 6 14 6 48 7 22 8 56 9 30      | <b>34</b> 9 4 9 38 10 12 10 46 11 20    |    |    |    |    |    |    |    |    |
| 6 25 7 0 7 35 8 10 8 45       | <b>35</b> 9 20 9 55 10 30 11 5 11 40    |    |    |    |    |    |    |    |    |
| 6 36 7 12 7 48 8 24 9 0       | <b>36</b> 9 36 10 12 10 48 11 24 12 0   |    |    |    |    |    |    |    |    |
| 6 47 7 24 8 1 8 38 9 15       | <b>37</b> 9 52 10 29 11 6 11 43 12 20   |    |    |    |    |    |    |    |    |
| 6 58 7 36 8 14 8 52 9 30      | <b>38</b> 10 8 10 46 11 24 12 2 12 40   |    |    |    |    |    |    |    |    |
| 7 9 7 48 8 27 9 6 9 45        | <b>39</b> 10 24 11 3 11 42 12 21 13 0   |    |    |    |    |    |    |    |    |
| 7 20 8 0 8 40 9 20 10 0       | <b>40</b> 10 40 11 20 12 0 12 40 13 20  |    |    |    |    |    |    |    |    |
| 7 31 8 12 8 53 9 34 10 15     | <b>41</b> 10 56 11 37 12 18 12 59 13 40 |    |    |    |    |    |    |    |    |
| 7 42 8 24 9 6 9 48 10 30      | <b>42</b> 11 12 11 54 12 36 13 18 14 0  |    |    |    |    |    |    |    |    |
| 7 53 8 36 9 19 10 2 10 45     | <b>43</b> 11 28 12 11 12 54 13 37 14 20 |    |    |    |    |    |    |    |    |
| 8 4 8 48 9 32 10 16 11 0      | <b>44</b> 11 44 12 28 13 12 13 56 14 40 |    |    |    |    |    |    |    |    |
| 8 15 9 0 9 45 10 30 11 15     | <b>45</b> 12 0 12 45 13 30 14 15 15 0   |    |    |    |    |    |    |    |    |
| 8 26 9 12 9 58 10 44 11 30    | <b>46</b> 12 16 13 2 13 48 14 34 15 20  |    |    |    |    |    |    |    |    |
| 8 37 9 24 10 11 10 58 11 45   | <b>47</b> 12 32 13 19 14 6 14 53 15 40  |    |    |    |    |    |    |    |    |
| 8 48 9 36 10 24 11 12 12 0    | <b>48</b> 12 48 13 36 14 24 15 12 36 0  |    |    |    |    |    |    |    |    |
| 8 59 9 48 10 37 11 26 12 15   | <b>49</b> 13 4 13 53 14 42 15 31 16 20  |    |    |    |    |    |    |    |    |
| 9 10 10 0 10 50 11 40 12 30   | <b>50</b> 13 20 14 10 15 0 15 50 16 40  |    |    |    |    |    |    |    |    |
| 9 21 10 12 11 3 11 54 12 45   | <b>51</b> 13 36 14 27 15 18 16 9 17 0   |    |    |    |    |    |    |    |    |
| 9 32 10 24 11 16 12 8 13 0    | <b>52</b> 13 52 14 44 15 36 16 28 17 20 |    |    |    |    |    |    |    |    |
| 9 43 10 36 12 29 12 22 13 15  | <b>53</b> 14 8 15 1 15 54 16 47 17 40   |    |    |    |    |    |    |    |    |
| 9 54 10 48 11 42 12 36 13 30  | <b>54</b> 14 24 15 18 16 12 17 6 18 0   |    |    |    |    |    |    |    |    |
| 10 5 11 0 11 55 12 50 13 45   | <b>55</b> 14 40 15 35 16 30 17 25 18 20 |    |    |    |    |    |    |    |    |
| 10 16 11 12 12 8 13 4 14 0    | <b>56</b> 14 56 15 52 16 48 17 44 18 40 |    |    |    |    |    |    |    |    |
| 10 27 11 24 12 21 13 18 14 15 | <b>57</b> 15 12 16 9 17 6 18 3 19 0     |    |    |    |    |    |    |    |    |
| 10 38 11 36 12 34 13 32 14 30 | <b>58</b> 15 28 16 26 17 24 18 22 19 20 |    |    |    |    |    |    |    |    |
| 10 49 11 48 12 47 13 46 14 45 | <b>59</b> 15 44 16 43 17 42 18 41 19 40 |    |    |    |    |    |    |    |    |
| 11 0 12 0 13 0 14 0 15 0      | <b>60</b> 16 0 17 0 18 0 19 0 20 0      |    |    |    |    |    |    |    |    |







|    | 51    | 52    | 53    | 54    | 55    |    | 56       | 57       | 58       | 59       | 60 |
|----|-------|-------|-------|-------|-------|----|----------|----------|----------|----------|----|
| 0  | 51 0  | 52 0  | 53 0  | 54 0  | 55 0  | 1  | 0 56 0   | 0 57 0   | 0 58 0   | 0 59 1   | 0  |
| 1  | 42 1  | 44 1  | 46 1  | 48 1  | 50 1  | 2  | 1 52 1   | 1 54 1   | 1 56 1   | 1 58 2   | 0  |
| 2  | 33 2  | 36 2  | 39 2  | 42 2  | 45 2  | 3  | 2 48 2   | 2 51 2   | 2 54 2   | 2 57 3   | 0  |
| 3  | 24 3  | 28 3  | 32 3  | 36 3  | 40 3  | 4  | 3 44 3   | 3 48 3   | 3 52 3   | 3 56 4   | 0  |
| 4  | 15 4  | 20 4  | 25 4  | 30 4  | 35 4  | 5  | 4 40 4   | 4 44 4   | 4 50 4   | 4 55 5   | 0  |
| 5  | 6 5   | 12 5  | 18 5  | 24 5  | 30 5  | 6  | 5 36 5   | 5 42 5   | 5 48 5   | 5 54 6   | 0  |
| 6  | 57 6  | 4 6   | 11 6  | 18 6  | 25 6  | 7  | 6 32 6   | 6 39 6   | 6 46 6   | 6 53 7   | 0  |
| 7  | 48 7  | 56 7  | 4 7   | 12 7  | 20 7  | 8  | 7 28 7   | 7 36 7   | 7 44 7   | 7 52 8   | 0  |
| 8  | 39 8  | 48 8  | 57 8  | 6 8   | 15 8  | 9  | 8 24 8   | 8 33 8   | 8 42 8   | 8 51 9   | 0  |
| 9  | 30 9  | 40 9  | 50 9  | 0 9   | 10 9  | 10 | 9 20 9   | 9 30 9   | 9 40 9   | 9 50 10  | 0  |
| 10 | 21 9  | 32 9  | 43 9  | 54 9  | 0 10  | 11 | 10 16 10 | 10 27 10 | 10 38 10 | 10 49 11 | 0  |
| 11 | 12 10 | 24 10 | 36 10 | 48 10 | 0 11  | 12 | 11 12 11 | 11 24 11 | 11 36 11 | 11 48 12 | 0  |
| 12 | 3 11  | 16 11 | 29 11 | 42 11 | 55 11 | 13 | 12 8 12  | 12 21 12 | 12 34 12 | 12 47 13 | 0  |
| 13 | 54 12 | 8 12  | 22 12 | 36 12 | 50 12 | 14 | 13 4 13  | 13 18 13 | 13 32 13 | 13 46 14 | 0  |
| 14 | 45 13 | 0 13  | 15 13 | 30 13 | 45 13 | 15 | 14 0 14  | 14 15 14 | 14 30 14 | 14 45 15 | 0  |
| 15 | 36 13 | 52 13 | 8 14  | 24 14 | 40 14 | 16 | 14 56 15 | 14 12 15 | 14 28 15 | 14 44 16 | 0  |
| 16 | 27 14 | 44 14 | 1 15  | 18 15 | 35 15 | 17 | 15 52 16 | 15 9 16  | 15 26 16 | 15 43 17 | 0  |
| 17 | 18 15 | 36 15 | 54 16 | 12 16 | 30 16 | 18 | 16 48 17 | 16 6 17  | 16 24 17 | 16 42 18 | 0  |
| 18 | 9 16  | 28 16 | 47 17 | 6 17  | 25 17 | 19 | 17 44 18 | 17 3 18  | 17 22 18 | 17 41 19 | 0  |
| 19 | 0 17  | 20 17 | 40 18 | 0 18  | 20 18 | 20 | 18 40 19 | 18 0 19  | 18 20 19 | 18 40 20 | 0  |
| 20 | 51 18 | 12 18 | 33 18 | 54 19 | 15 19 | 21 | 19 36 19 | 19 57 20 | 19 18 20 | 19 39 21 | 0  |
| 21 | 42 19 | 4 19  | 26 19 | 48 20 | 10 20 | 22 | 20 32 20 | 20 54 21 | 20 16 21 | 20 38 22 | 0  |
| 22 | 33 19 | 56 20 | 19 20 | 42 21 | 5 21  | 23 | 21 28 21 | 21 51 22 | 21 14 22 | 21 37 23 | 0  |
| 23 | 24 20 | 48 21 | 12 21 | 36 22 | 0 22  | 24 | 22 24 22 | 22 48 23 | 22 12 23 | 22 36 24 | 0  |
| 24 | 15 21 | 40 22 | 5 22  | 30 23 | 59 23 | 25 | 23 20 23 | 23 45 24 | 23 10 24 | 23 35 25 | 0  |
| 25 | 6 22  | 32 23 | 58 23 | 24 24 | 50 24 | 26 | 24 16 24 | 24 42 25 | 24 8 25  | 24 34 26 | 0  |
| 26 | 57 23 | 24 23 | 51 24 | 18 24 | 45 24 | 27 | 25 12 25 | 25 39 26 | 25 6 26  | 25 33 27 | 0  |
| 27 | 48 24 | 16 24 | 44 25 | 12 25 | 40 25 | 28 | 26 8 26  | 26 36 27 | 26 4 27  | 26 32 28 | 0  |
| 28 | 39 25 | 8 25  | 37 26 | 6 26  | 35 26 | 29 | 27 4 27  | 27 33 28 | 27 2 28  | 27 31 29 | 0  |
| 29 | 30 26 | 0 26  | 30 27 | 0 27  | 30 27 | 30 | 28 0 28  | 28 30 29 | 28 0 29  | 28 30 30 | 0  |

|    | 51    | 52    | 53    | 54    | 55    |    | 56       | 57       | 58    | 59    | 60 |
|----|-------|-------|-------|-------|-------|----|----------|----------|-------|-------|----|
| 26 | 21 26 | 52 27 | 23 27 | 54 28 | 25 28 | 31 | 28 56 29 | 27 27 29 | 58 30 | 29 31 | 0  |
| 27 | 12 27 | 44 28 | 16 28 | 48 29 | 20 29 | 32 | 29 52 30 | 24 30    | 56 31 | 28 32 | 0  |
| 28 | 3 28  | 36 29 | 9 29  | 42 30 | 15 30 | 33 | 30 48 31 | 21 31    | 54 32 | 27 33 | 0  |
| 29 | 54 29 | 28 30 | 7 30  | 36 31 | 10 31 | 34 | 31 44 32 | 18 32    | 52 33 | 26 34 | 0  |
| 30 | 45 30 | 20 30 | 59 31 | 30 32 | 5 32  | 35 | 32 40 33 | 15 33    | 50 34 | 25 35 | 0  |
| 31 | 36 31 | 12 31 | 48 32 | 24 33 | 0 33  | 36 | 33 36 34 | 12 34    | 48 35 | 24 36 | 0  |
| 32 | 27 32 | 4 32  | 41 33 | 18 34 | 55 34 | 37 | 34 32 35 | 9 35     | 46 36 | 23 37 | 0  |
| 33 | 18 33 | 56 34 | 24 34 | 12 35 | 50 35 | 38 | 35 28 36 | 6 36     | 44 37 | 22 38 | 0  |
| 34 | 9 34  | 48 35 | 27 35 | 6 36  | 45 36 | 39 | 36 24 37 | 3 37     | 42 38 | 21 39 | 0  |
| 35 | 0 35  | 40 36 | 20 36 | 0 37  | 40 37 | 40 | 37 20 38 | 0 38     | 40 39 | 20 40 | 0  |
| 36 | 51 36 | 32 37 | 13 37 | 54 38 | 35 38 | 41 | 38 16 39 | 57 39    | 38 40 | 19 41 | 0  |
| 37 | 42 37 | 24 38 | 6 38  | 48 39 | 30 39 | 42 | 39 12 40 | 54 40    | 36 41 | 18 42 | 0  |
| 38 | 33 38 | 16 39 | 59 40 | 42 41 | 25 41 | 43 | 40 8 41  | 51 41    | 34 42 | 17 43 | 0  |
| 39 | 24 39 | 8 40  | 52 41 | 36 42 | 20 42 | 44 | 41 4 42  | 48 42    | 32 43 | 16 44 | 0  |
| 40 | 15 40 | 0 41  | 45 42 | 30 43 | 15 43 | 45 | 42 0 43  | 45 43    | 30 44 | 15 45 | 0  |
| 41 | 6 41  | 54 42 | 42 43 | 28 44 | 10 44 | 46 | 43 56 44 | 42 44    | 28 45 | 14 46 | 0  |
| 42 | 57 42 | 44 43 | 31 44 | 18 45 | 5 45  | 47 | 44 52 45 | 39 45    | 26 46 | 13 47 | 0  |
| 43 | 48 43 | 36 44 | 24 45 | 12 46 | 0 46  | 48 | 45 48 46 | 36 46    | 24 47 | 12 48 | 0  |
| 44 | 39 44 | 28 45 | 17 46 | 6 47  | 55 47 | 49 | 46 44 47 | 33 47    | 22 48 | 11 49 | 0  |
| 45 | 30 45 | 20 46 | 10 47 | 0 48  | 50 48 | 50 | 47 40 48 | 30 48    | 20 49 | 10 50 | 0  |
| 46 | 21 46 | 12 47 | 3 48  | 54 49 | 45 49 | 51 | 48 36 49 | 27 49    | 18 50 | 9 51  | 0  |
| 47 | 12 47 | 4 48  | 26 49 | 48 50 | 10 50 | 52 | 49 32 50 | 24 50    | 16 51 | 8 52  | 0  |
| 48 | 3 48  | 36 50 | 16 50 | 42 51 | 5 51  | 53 | 50 28 51 | 21 51    | 14 52 | 7 53  | 0  |
| 49 | 54 49 | 28 51 | 7 51  | 36 52 | 30 52 | 54 | 51 24 52 | 18 52    | 12 53 | 6 54  | 0  |
| 50 | 45 50 | 20 52 | 59 53 | 30 54 | 25 54 | 55 | 52 20 53 | 15 53    | 10 54 | 5 55  | 0  |
| 51 | 36 51 | 12 53 | 48 54 | 24 55 | 0 55  | 56 | 53 16 54 | 12 54    | 8 55  | 4 56  | 0  |
| 52 | 27 52 | 4 54  | 41 55 | 18 56 | 55 56 | 57 | 54 12 55 | 9 55     | 6 56  | 3 57  | 0  |
| 53 | 18 53 | 56 55 | 24 55 | 12 56 | 50 56 | 58 | 55 8 56  | 6 56     | 4 57  | 2 58  | 0  |
| 54 | 9 54  | 48 56 | 27 56 | 6 57  | 45 57 | 59 | 56 4 57  | 3 57     | 2 58  | 1 59  | 0  |
| 55 | 0 55  | 40 57 | 20 57 | 0 58  | 40 58 | 60 | 57 0 58  | 0 59     | 0 60  | 0 60  | 0  |