Chebyshev's continuous adding machine

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Abstract

This article analyzes the structure of Chebyshev's continuous adding machine, the readability of its values, and its sensitivity to backlash.

1 Discrete adding machines

The first calculating machines were discrete machines. These machines had a number of states, corresponding to clearly identified numerical values. The passage from one state to another was obtained by various means, but it was always meant to be quick. In such machines, digits were usually represented by the positions of wheels, and sometimes one wheel would serve for several digits. Figure 1 shows such a display, with ten wheels which have been laid out flat, or unrolled. Each vertical strip represents one wheel, and a wheel may sometimes contain more than one series of integers 0 to 9.

In such a machine, adding one to the units will advance that wheel by one position, and in case the figure displayed was 9, it becomes 0, and at the same time (or shortly afterwards) the second wheel is advanced by one unit, and so on. These machines have to take care of carries, and this is often a source of problems or limitations. Many of these machines can for instance not work backwards.

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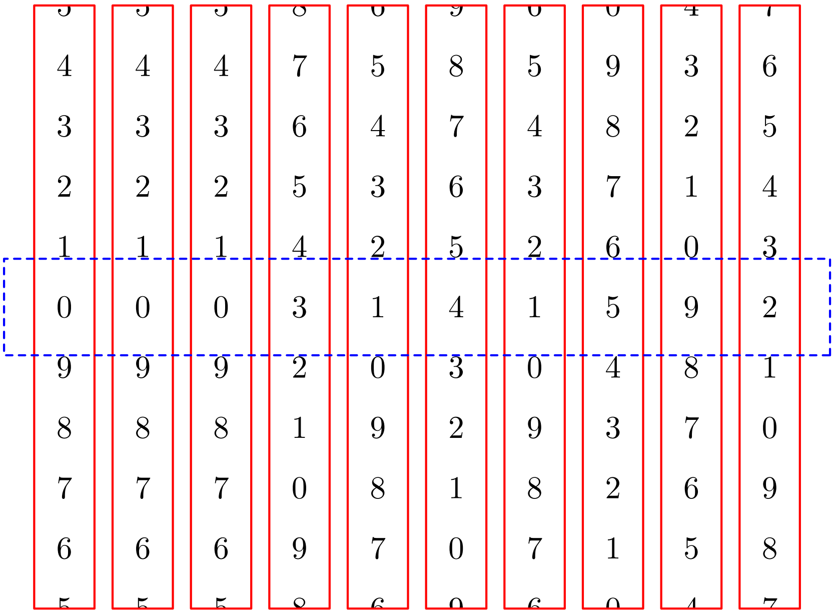


Figure 1: The display of a discrete value. The digits are aligned on horizontal lines and there is a special line (here marked by a dashed rectangle) for reading the value, which is here 0003141592.

2 Chebyshev's continuous adding machine

In the 1870s, the Russian mathematician Pafnuty Lvovich Chebyshev (Пафнутий Львович Чебышёв) (1821–1894), who had been interested in the theory of mechanisms,[1] came up with a new construction for adding numbers. He first constructed a machine for additions only, and this machine is now kept in St Petersburg. This machine was probably exhibited at the scientific meeting attended by Chebyshev in Clermont-Ferrand in 1876. In 1882, Chebyshev returned to France and had a second machine built in Paris by the firm Gautier [3], [18, p. 403]. The French mathematician Édouard Lucas [7, 8] arranged that this machine be "temporarily" lent to the *Conservatoire des Arts et Métiers* in Paris, probably during Chebyshev's next trip in 1884, but it was donated to the museum only at the end of Chebyshev's last trip to France in 1893. During that visit, Chebyshev gave d'Ocagne all the necessary explanations on the machine, and he provided a description which was published in d'Ocagne's *Le calcul simplifié* [15]. According to d'Ocagne, another copy of the machine was built after Chebyshev's death for exhibition in Moscow [4, 17].

Chebyshev's idea is based on the following observations. In a discrete machine, the position of a given wheel, say that of the hundreds, does not show exactly how many hundreds there are in a certain number, but only how many *complete* hundreds there are. For instance, in the above example, 592 means that there are five complete hundreds, even though the number is much closer to 600 than to 500. Now, Chebyshev considered a new display in which each figure would move regularly, at the same time as the units. The tens would gradually move from one position to the next, and not instantly skip when the units go from 9 to 0. It is therefore necessary to have a construction such that the wheel of the tens moves 10 times slower than that of the units, the wheel of the hundreds moves 10 times slower than that of the tens, and so on. This can be achieved with gears in a 1:10 ratio between the units and the tens, between the tens and the hundreds, and so on. This solution then totally dispenses with the usually complex mechanisms required for an intermittent motion. It also makes it possible to do calculations much faster.

However, such a machine will not be used to add only units. It must be possible to add tens, hundreds, etc. separately, and in any order. Adding one hundred, for instance, would then cause the wheel of the tens to move

[1] An overview of Chebyshev's mechanisms and several computer reconstructions and animations, as well as many documents related to these mechanisms, can be found on `http://tcheb.ru`.

by 10 places, and the unit wheel to move by 100 places. This is of course quite impractical. On the other hand, adding 10 places to the tens, or 100 places to the units, does in fact not change their values, so that a better solution would be to keep these wheels still when adding places to the hundreds. In other words, what is desired is that the addition of some units to a given position only moves that wheel, and those on its left, but not the wheels on its right.

A solution to this problem involves additional wheels between the figure wheels. These additional wheels will be used whenever a value has to be added at a specific position, and changes have to be propagated towards the left, but not towards the right. Figure 2 shows Chebyshev's solution. The central arbor is X and around it there are a number of freely revolving blocks. Each block is made of a figure wheel (n_1 and n_2 in the figure), and of two gears (d and f on each side of n_1). These three elements always move together.

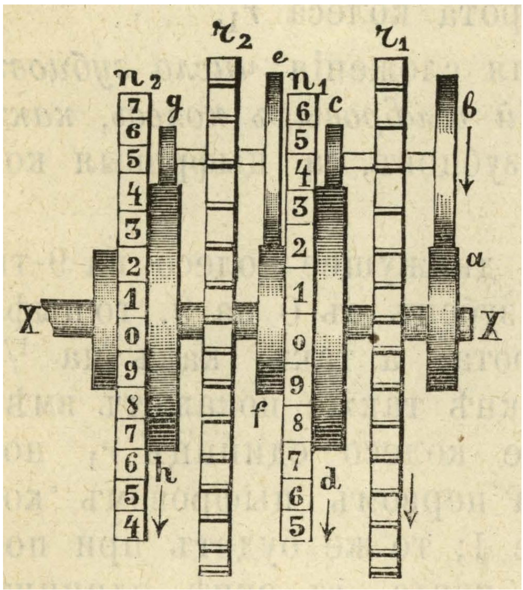


Figure 2: Chebyshev's wheels. (excerpt from [20])

At the right, there is one gear a which is fixed on the arbor X and not moving. Moreover, at the right of the first figure wheel, and between each two blocks, there is an additional disk R (r_1 and r_2 on the figure), also moving freely on the arbor X. This disk is the carrier for two satellite wheels (b and c, and e and g), which are used to transmit the motion from one block to the next.

If the carriers r_1 and r_2 do not move, whenever n_1 moves, it will cause e to rotate, therefore g, and therefore h, and the figure wheel n_2 will advance. The ratios can be chosen so that n_2 advances 10 times slower than n_1. But of course, our assumption is slightly incorrect, in that n_1 cannot just move by itself and a and r_1 remain still. n_1 can only move if the carrier r_1 moves. And that is in fact how units are added. The user will not advance the figure wheel n_1, but he/she will advance the carrier r_1, which is equipped with notches. One notch in r_1 should correspond to one unit of n_1, and we will see in a moment how this is done.

Consequently, moving the carrier r_1, and keeping all the other carriers still will cause n_1 to advance by a number of units, n_2 to advance ten times less, n_3 to advance 100 times less, and so on.

One may now wonder what happens if we move the carrier r_2, keeping all the other carriers still. What happens is very simple to understand. Since the wheel a is not moving, the first figure wheel will not move, and therefore the gear f will be still. Moving r_2 with gear f still will be similar to moving r_1 with gear a still. Everything will behave as if the tens had become the units, and the motion will only be propagated towards the left. All this rests on two assumptions, that the first gear a is never moving, and that the motion of a figure wheel cannot move a carrier.

Figure 3 shows a side view of the wheels for the transmission from one figure to the one to its left. The wheels are named like in figure 2. X is the central arbor, and A is the satellite arbor, moving on plate R (the "receiving" wheel). When R moves, b will roll on a, this will rotate c, which in turn will rotate d, to which the next figure wheels are attached. In turn, the rotation of d will be transmitted further left, but without any rotation of the receiving wheels, because these are kept in place by springs.

If we assume that a is not moving, we have to find out what is the effect of the rotation of R on wheel d. Let us assume that R turns by the angle α_R, for instance clockwise (the actual direction is irrelevant, as long as we stick to the same convention), and that a is not moving. We also use $\alpha'_{\dots}$ for angles in R's reference frame, and $\alpha_{\dots}$ for angles in an absolute reference frame. Then, in R's reference frame,

$$\alpha'_a = -\alpha_R \tag{1}$$

$$\alpha'_b = -\frac{a}{b}\alpha'_a = \frac{a}{b}\alpha_R = \alpha'_c \tag{2}$$

$$\alpha'_d = -\frac{c}{d}\alpha'_c = -\frac{ac}{bd}\alpha_R \tag{3}$$

Consequently, in the absolute reference frame, d has turned by the angle

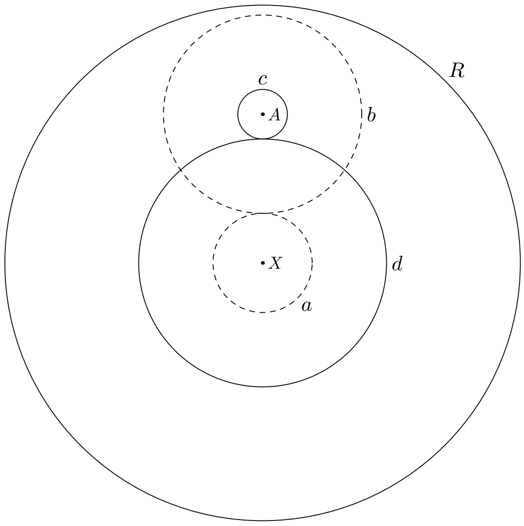


Figure 3: A block of wheels seen from the side.

$$\alpha_d = \alpha_R + \alpha'_d \tag{4}$$
$$= \alpha_R - \frac{ac}{bd}\alpha_R = \alpha_R\left(1 - \frac{ac}{bd}\right) \tag{5}$$

With the values of Chebyshev's second adding machine, this gives:

$$\alpha_d = \alpha_R\left(1 - \frac{24 \times 12}{48 \times 60}\right) \tag{6}$$
$$= \alpha_R\left(1 - \frac{1}{10}\right) = \frac{9}{10}\alpha_R \tag{7}$$

with $a = 24$, $b = 48$, $c = 12$ and $d = 60$.

R contains 27 notches (and not numbers), so that advancing R by one notch corresponds to $\alpha_R = \frac{360}{27}^{\circ}$, and $\alpha_d = \frac{9}{10} \times \frac{360}{27}^{\circ} = 12^{\circ}$, that is, exactly one digit, assuming that the figure wheels contain three times the interval 0–9.

Moreover, d has turned in the same direction as the receiving wheel. This is consistent with Chebyshev's second adding machine, and figure 8 shows that the figures increase from bottom to top (for instance 8 and 9 in the window left), and adding a digit is obtained by turning a receiving wheel towards the bottom by a certain number of units.

However, on Chebyshev's first adding machine (figure 11), the figures increase towards the bottom. It is likely that the ratio of the wheels is different than the one given above. The motion could indeed be reversed if $\frac{ac}{bd} = \frac{19}{10}$, for instance with $a = 48$, $b = 24$, $c = 38$, and $d = 40$. Unfortunately, we do not know the details of Chebyshev's machine, but hopefully this can be clarified in the future, or perhaps in the archives of the St Petersburg museum.

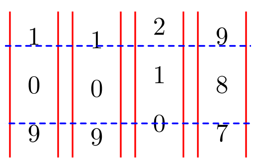
3 Configurations

With discrete displays, we never think about the alignment of figures, because this seems to be an obvious matter. Although figures briefly move from one value to the next, they appear most of the time at integer positions. It is therefore natural to align the figures, initially and at all times, and the reader may certainly be puzzled by this consideration. In such a display, a value may start with 0000, then move to 0001, then 0002, etc., then 0009, then 0010, and so on. The initial 0's appear on a same line. They could of course be aligned differently, but that would obviously not be very convenient, as it is desirable to be able to read the values as clearly as possible.

But with a continuous (opposed to intermittent) display, several figures move at the same time. When the units advance by one unit, the tens advance by a tenth, the hundreds advance by a hundredth, and so on. Therefore, even if we start with 0000, the figures will little by little be offset, and no longer be aligned. Figure 4 show a 4-digit display for the values 0 to 11. When the units go from 0 to the next 0, the tens slowly move from 0 to 1, and the hundreds also move, but even slowlier.

3.1 The issue of readability

A continuous display raises a readability question, as



should be read 0008 and not as 0018, which it seems to display. There seems therefore to be an ambiguity, and that such a continuous display is unusable. But is it really?

In fact, the above value can be read without ambiguity, but it has to be read in another way than the way we may find most natural. Although the

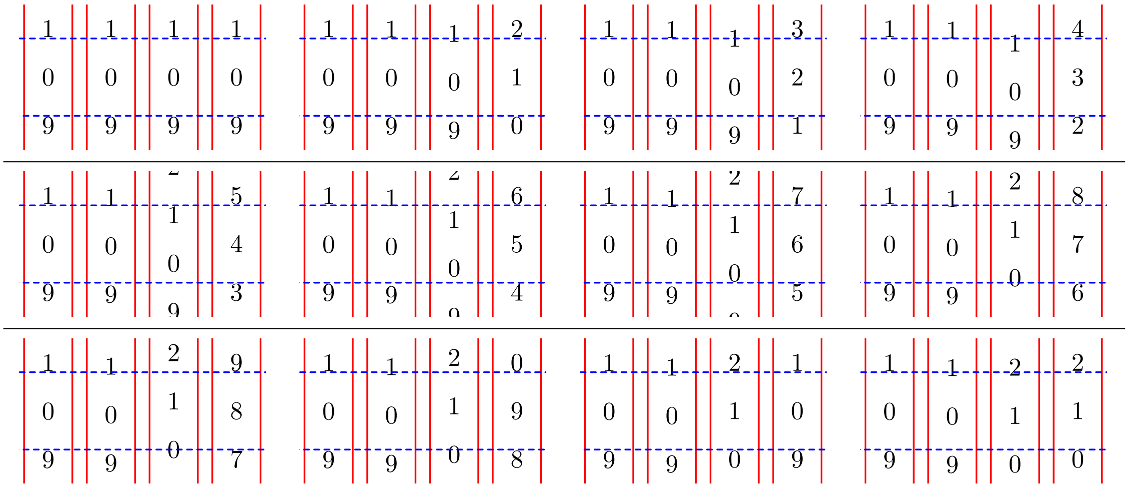
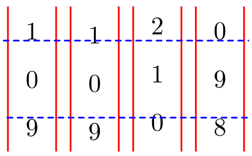


Figure 4: The first values of a continuous display when starting at 0000.

tens are close to 1, and might even show exactly 1 if there is some backlash, we should not consider this value as the value to be read. So, how can we read unambigously such a display?

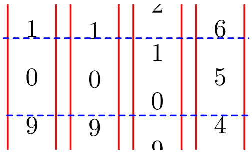
Answering this question seems even more difficult when we come closer to the threshhold:



There is in fact a very simple algorithm for reading these values. Let us denote V_i the real value of a digit wheel, and v_i the value that is *read*. We have of course $0 \leq V_i, v_i < 10$, $V_i \in \mathbb{R}$ and $v_i \in \mathbb{N}$. Now, the algorithm starts with the units which are assumed to be an integer. The value shown is the value to be retained. In the latter case, the value of the units is $v_0 = V_0 = 9$. Now, whenever the units are close to 0 or 10, the tens are close to an integer, and we can unambiguously know the value of this integer. In that case, it is $w_1 = 1$. But w_1 is neither the real value V_1 of the tens, nor the value v_1 to be read. Now, in order to find v_1, the value to retain for the tens, we can distinguish two cases:

- if $v_0 > 8$, then $v_1 = w_1 - 1$;
- if $v_0 < 2$ then $v_1 = w_1$.

Of course, this will only work in the previous example and when v_0 is close to 0 or 10, and also only when $w_1 > 0$, but we will be able to generalize it easily. Before we do that, we can look at what happens when the figures of the tens are halfway between two integers:



The first thing to observe here is that the display is in fact much less ambiguous than above. We see right away that the tens are neither 0 or 1, or rather, that the value to be retained will be either 0 or 1. (Our sentence seems contradictory, because some of our words are intentionally ambiguous.) In that case, the algorithm also is rather simple: if $1 < v_0 < 9$, v_1 is the integer part of the fractional value displayed.

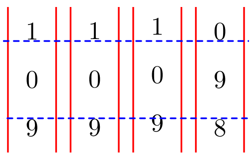
Summing up, we have the following algorithm, where V_i is the real value of figure i, v_i is the value *read*, $E(x)$ represents the integer part of x, and $R(x) = E(x + 0.5)$ the rounded value of x:

- $v_0 \leftarrow R(V_0) \bmod 10$
- for $i > 0$:
  - if $v_{i-1} > 7$ then $v_i \leftarrow (R(V_i) - 1) \bmod 10$
  - if $v_{i-1} < 3$ then $v_i \leftarrow R(V_i) \bmod 10$
  - otherwise $v_i \leftarrow E(V_i)$

This can also be expressed as follows:

- $v_0 \leftarrow E(V_0 + 0.5) \bmod 10$
- $v_{i+1} \leftarrow \begin{cases} (E(V_{i+1} + 0.5) - 1) \bmod 10 & \text{if } v_i > 7 \\ E(V_{i+1} + 0.5) \bmod 10 & \text{if } v_i < 3 \\ E(V_{i+1}) & \text{otherwise} \end{cases}$

The following configuration



is then *unambiguously* read as 9999, and not as 0009.

This will remain true even if the figure wheels do slightly move, as long as they do not move too much. Now, what is *too much*? As it must be apparent, the important features of the reading algorithm are the notions of being close to an integer, and of being far from an integer. The backlash of the wheels must not be such as being able to move the tens (or any other wheel) from 0 to 0.5 for instance, for it would then be genuinely difficult to know which integer is represented by the tens when the units were about to pass from 9 to 0. Each figure should certainly not depart more than 0.2 or 0.3 units from its theoretical value, and we can for instance hope for a display error of V_i of no more than 0.1 units of that wheel. Given these assumptions, we have in fact a quite robust algorithm.

We represent the theoretical[2] values of the figure wheels as the *configuration* $\langle V_p, V_{p-1}, \ldots, V_1, V_0 \rangle_0$, the subscript 0 being a reminder that we have started with 000…00. We can then speak of "0-configurations." The actual values (due to backlash) are represented by $\langle W_p, W_{p-1}, \ldots, W_1, W_0 \rangle_0$. And the values read are $\langle v_p, v_{p-1}, \ldots, v_1, v_0 \rangle_0$.

There is a relationship between a value N and its representation as a configuration $\langle V_p, V_{p-1}, \ldots, V_1, V_0 \rangle_0$. We must have $V_p = (N/10^p) \bmod 10$, $V_{p-1} = (N/10^{p-1}) \bmod 10$, …, $V_1 = (N/10) \bmod 10$, and $V_0 = N \bmod 10$. For instance, 157 is represented by $\langle 1.57, 5.7, 7 \rangle_0$, and not by $\langle 1, 5, 7 \rangle_0$.

Conversely, a given configuration corresponds to a certain value N. We merely have $N \bmod 10^{p+1} = 10^p V_p$.

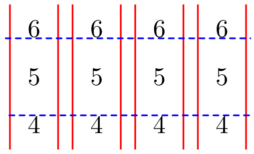
The robustness of the algorithm means that the actual configurations $\langle 9.98, 9.3 \rangle_0$ and $\langle 0.1, 9.3 \rangle_0$ may both denote the integer 99, so that there is a certain insensitivity to backlash. We will return to the influence of backlash in section 6.

3.2 Chebyshev's initial values

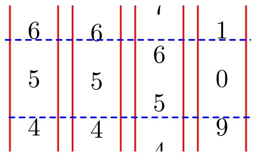
3.2.1 5-configurations

Starting with aligned zeros (0-configurations) may seem the most natural initial choice, but it is not the only one. In his adding machine, Chebyshev chose a setting in which the figures half way between 0 and 10 are aligned, namely a setting in which the fives are aligned. Such a configuration will be called a 5-configuration:

[2]By "theoretical," as alluded to previously, we mean that the influence of backlash is neglected.



In that setting, when the units reach 0, the tens are not close to an integer, but right in between:



Such a configuration is not ambiguous, and like above, we merely have to chose the upper value of the tens if the (read) units are 0 or above, and the lower value of the tens if the (read) units are below 0. And if the (read) units are close to 5, the tens will be close to an integer, and that integer will be the one to retain for the tens.

If we define two congruent configurations as two configurations which are reachable one from the other by a normal rotation (without disassembly) of the wheels, then it appears that $\langle 5, 5, \ldots, 5, 5\rangle_5$ is not congruent to $\langle 0, 0, \ldots, 0, 0\rangle_5$. It is easy to see that $\langle 5, 5, 5, 5\rangle_5 \equiv \langle 4, 5, 5, 5\rangle_5$ because any addition of 1 to the thousands corresponds to the addition of 10 to the hundreds, and so on, and therefore does not alter the figure of the hundred. So $\langle 5, 5, 5, 5\rangle_5 \equiv \langle 4, 5, 5, 5\rangle_5 \equiv \ldots \equiv \langle 0, 5, 5, 5\rangle_5$. But now, any smaller value will break the alignment of the initial 0. In fact, the first 0 is only at the same position as the figures of $\langle 5, 5, 5, 5\rangle_5$ when the value reads $\langle 0, 5, 5, 5\rangle_5$. For other values, the first figure 0 will be shifted up or down, and therefore the value $\langle 0, 0, 0, 0\rangle_5$ is not reachable. What this means, of course, is not that there is no value 0, but that the value 0 is not displayed as $\langle 0, 0, 0, 0\rangle_5$, if we take $\langle 5, 5, 5, 5\rangle_5$ as an initial value.

Every valid theoretical configuration represents some value, and we may ask which configuration represents the value 0? This question is easy to answer, and it suffices to subtract 55…55 to the configuration $\langle \ldots, 5, 5, 5, 5, 5\rangle_5$. We first subtract 5, and we obtain the configuration

$$\langle \ldots, 4.9995, 4.995, 4.95, 4.5, 0\rangle_5 \,.$$

Then we subtract 50, and we obtain the configuration

$$\langle \ldots, 4.9945, 4.945, 4.45, 9.5, 0\rangle_5 \,.$$

We continue by subtracting 500, and we obtain the configuration

$$\langle \dots, 4.9445, 4.445, 9.45, 9.5, 0 \rangle_5 .$$

And so on, so that the value representing 0 is actually

$$Z_0 = \langle 9.4 \dots 45, \dots, 9.445, 9.45, 9.5, 0 \rangle_5 .$$

This assumes that $\langle 5, 5, 5, 5 \rangle_5$ represents the value 5555. But if we instead considered that $\langle 5, 5, 5, 5 \rangle_5$ represents the value 5555.5, then the value representing 0 would actually be

$$Z'_0 = \langle 9.4 \dots 45, \dots, 9.445, 9.45, 9.5 \rangle_5$$

which is admittedly more regular. We will however stick to the somewhat irregular interpretation in which 0 is represented by Z_0.

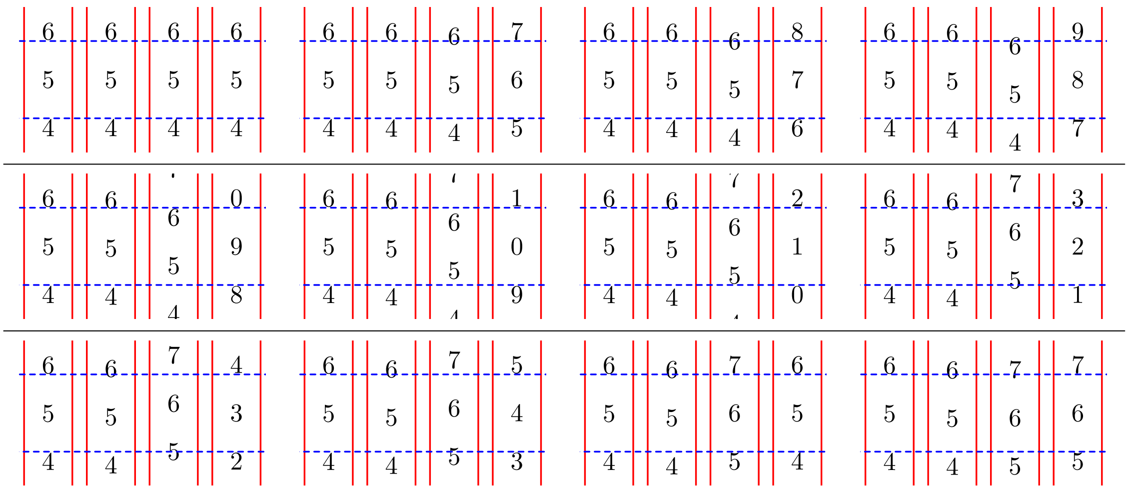


Figure 5: The first values when starting with aligned 5s.

3.2.2 From a value to a 5-configuration

Given a value n, we may want to know which 5-configuration represents it. All we have to do is add n to the value V_0 of Z_0, $n/10$ to the value V_1 of Z_0, $n/100$ to the value V_2 of Z_0, and so on. Eventually, we have

$$\begin{aligned} C &= \left\langle \dots, \left(9.445 + \frac{n}{1000}\right) \bmod 10, \left(9.45 + \frac{n}{100}\right) \bmod 10, \left(9.5 + \frac{n}{10}\right) \bmod 10, n \bmod 10 \right\rangle_5 \\ &= \left\langle \dots, \left(\frac{9445 + n}{1000}\right) \bmod 10, \left(\frac{945 + n}{100}\right) \bmod 10, \left(\frac{95 + n}{10}\right) \bmod 10, n \bmod 10 \right\rangle_5 \end{aligned}$$

3.2.3 From a 5-configuration to a value

Given a valid 5-configuration $Z = \langle V_p, V_{p-1}, \ldots, V_1, V_0 \rangle_5$, we may also want to find out which value it represents, and we want to do it in a robust way.

The 5-configurations will behave like the 0-configurations, merely shifted by the configuration $\langle -0.55\ldots 5, \ldots, -0.55, -0.5, 0 \rangle_5$. We can therefore derive the reading algorithm:

- $v_0 \leftarrow E(V_0 + S(0) + 0.5) \bmod 10$
- $v_{i+1} \leftarrow \begin{cases} (E(V_{i+1} + S(i+1) + 0.5) - 1) \bmod 10 & \text{if } v_i > 7 \\ E(V_{i+1} + S(i+1) + 0.5) \bmod 10 & \text{if } v_i < 3 \\ E(V_{i+1} + S(i+1)) \bmod 10 & \text{otherwise} \end{cases}$

where $S(i) = \frac{5(10^i - 1)}{9 \cdot 10^i}$, that is, $0.55\ldots 5$, with a total of i figures 5.

For instance, with the configuration $\langle 5, 5, 5, 5 \rangle_5$, we obtain $v_0 = 5$, $v_1 = E(5 + 0.55) = 5$, $v_2 = E(5 + 0.555) = 5$, and $v_3 = E(5 + 0.5555) = 5$.

The above algorithm translates graphically (figure 6):

- if v_i is close to 5, then the next figure is close to an integer, and its value can be obtained by rounding it to the nearest integer; v_i can therefore point horizontally to the value immediately on the left;
- if v_i if close to 10, but below 10, then the next figure is half way between two integers, and the lowest one should be taken; consequently, values of v_i above 5 should point towards this lowest value, and therefore point somewhat down, the more down that they are close to 10;
- finally, if v_i is close to 0, but above 0, then the next figure is also half way between two integers, but the upper one should be taken; consequently, values of v_i below 5 should point towards this upper value, and therefore point somewhat up, the more up that they are close to 0.

Similarly, when we start with aligned 0s, the algorithm can also be expressed graphically (figure 7).

This is exactly what Chebyshev does in the addition component of his second calculating machine. Chebyshev writes that the figures are written on white strips and that these strips can easily be followed from the right to the left [18, p. 403].

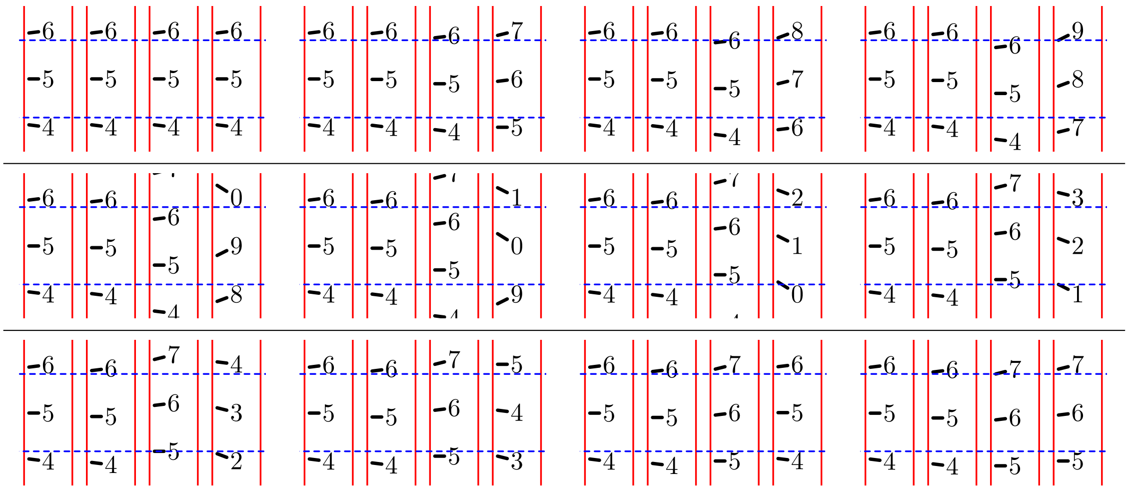


Figure 6: The first values when starting at 5, with reading hints.

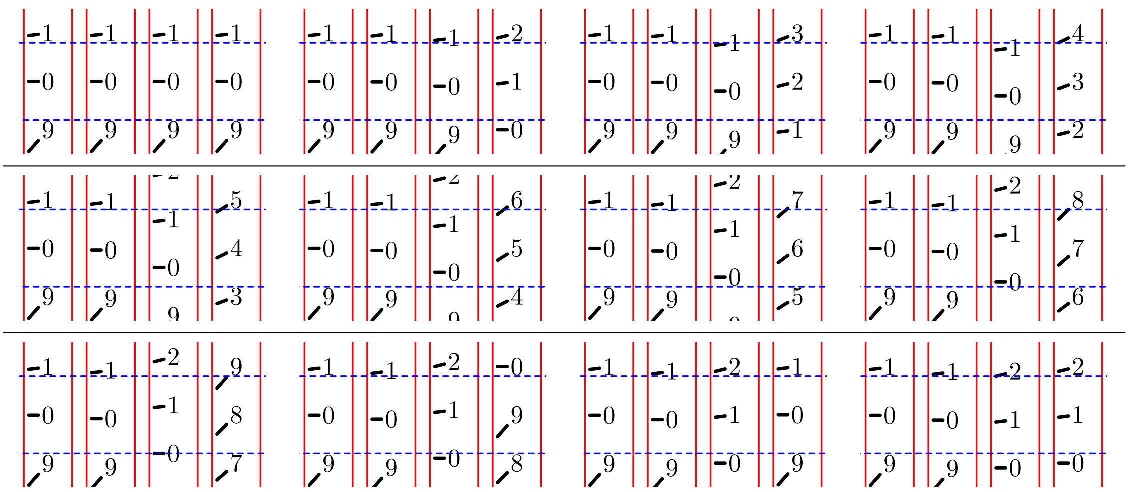


Figure 7: The first values when starting at 0, with reading hints.

4 Existing machines

As mentioned previously, the *Conservatoire des Arts et Métiers* in Paris holds the machine that Chebyshev had built in 1882. The museum also holds an epicycloidal gear train illustrating the ratio from the units to the tens. This second machine seems to be mounted correctly. It is possible that it has never been disassembled since the Chebyshev's last visit to Paris in 1893.

On the other hand, Chebyshev's first calculating machine is located at St Petersburg. The pictures of the machine seem to indicate that it is not correctly mounted (see figure 11). An animation has been made of that machine (see `http://tcheb.ru`), but the animation assumes incorrectly that the 0s are initially aligned.

It is possible that another copy of one of the machines is located in Moscow.

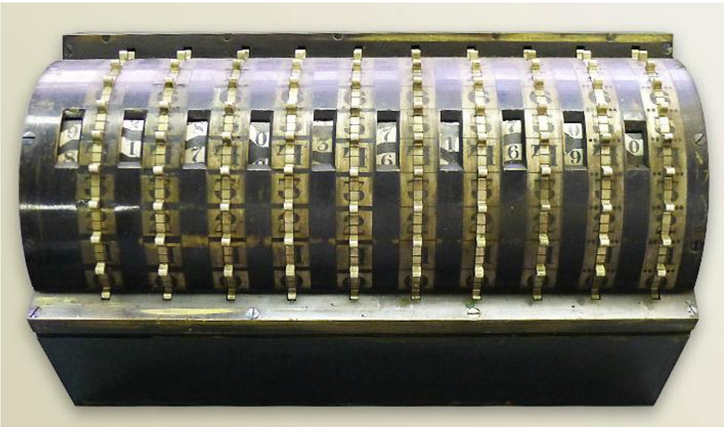


Figure 8: The addition component of Chebyshev's second calculating machine. The machine seems to be mounted correctly. For instance, the units show 0, but the tens are half way between 0 and 9. (picture from `http://tcheb.ru`)

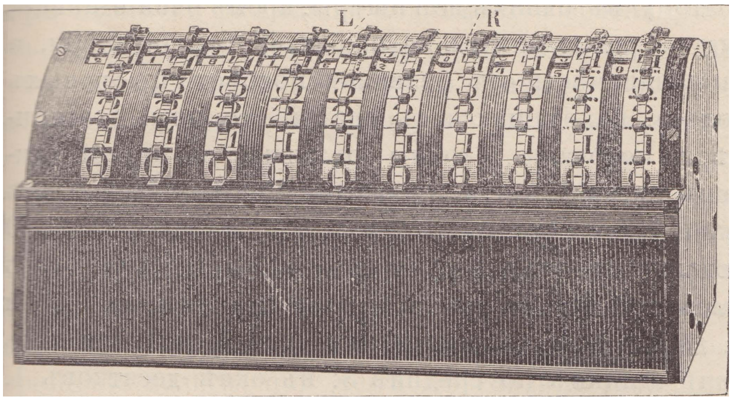


Figure 9: The addition component of Chebyshev's second calculating machine. (excerpt from [19]).

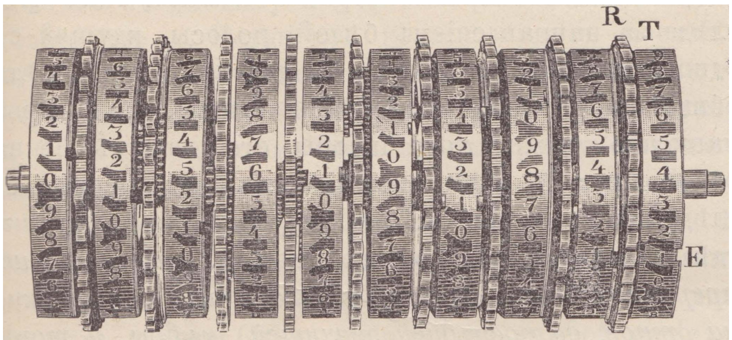


Figure 10: Chebyshev's wheels for his second calculating machine. (excerpt from [19])

Figure 11: Detail of Chebyshev's first calculating machine. This machine is not correctly mounted. When a figure is 0, the next one should be between two digits, and this isn't the case for the units and tens. Moreover, because the mounting is incorrect, it is difficult to follow the white strips. The first 0 points downwards, but there is no figure downwards in the tens. The third figure is 3, and the next one should then be almost centered, which isn't the case. (picture from `http://tcheb.ru`)

5 Addition examples

The following figures show a number of examples for the addition of two numbers. Of course, subtraction is similar and need not be detailed.

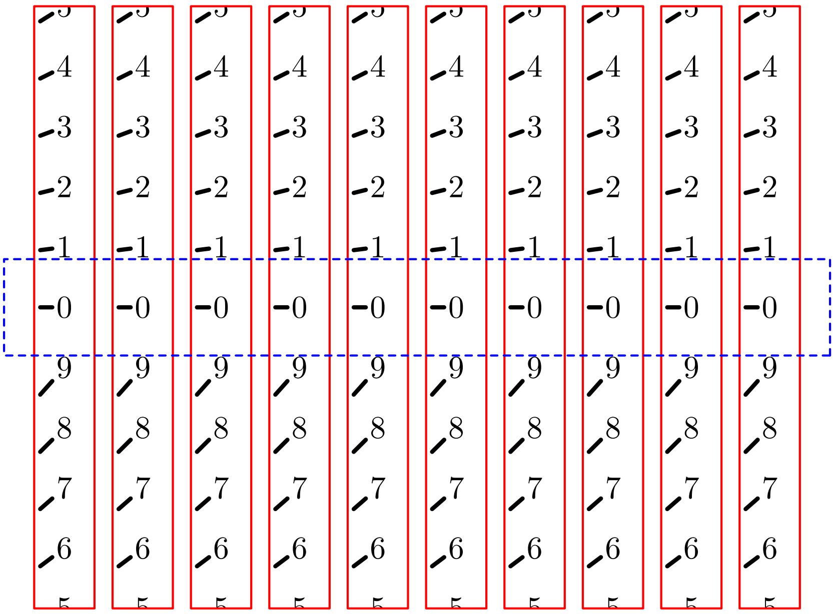


Figure 12: Initial configuration with aligned zeroes.

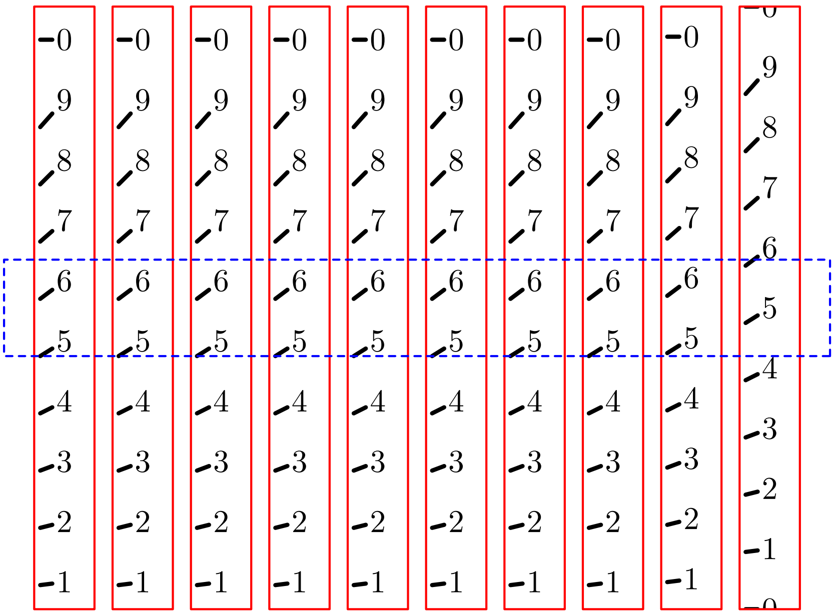


Figure 13: The representation of 5555555555 in a 0-configuration.

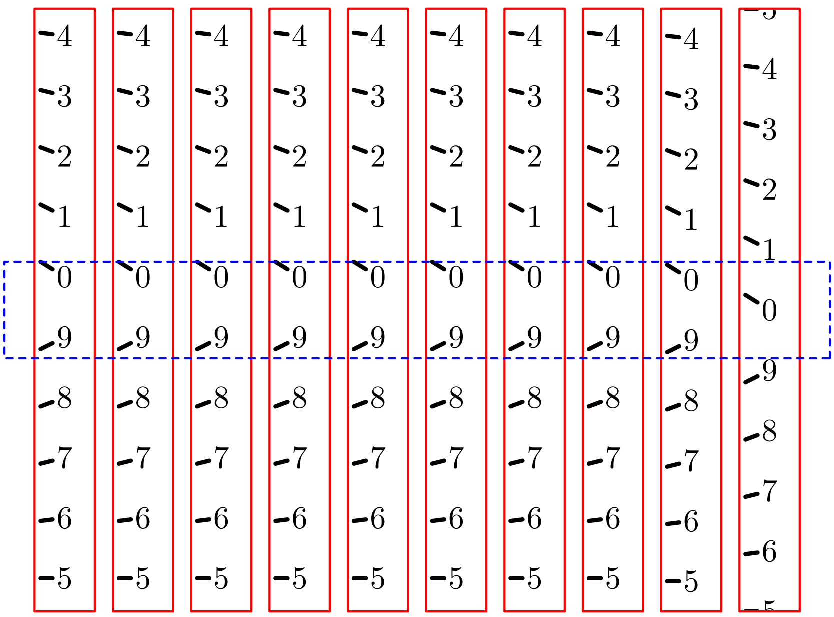


Figure 14: The representation of 0 in a 5-configuration.

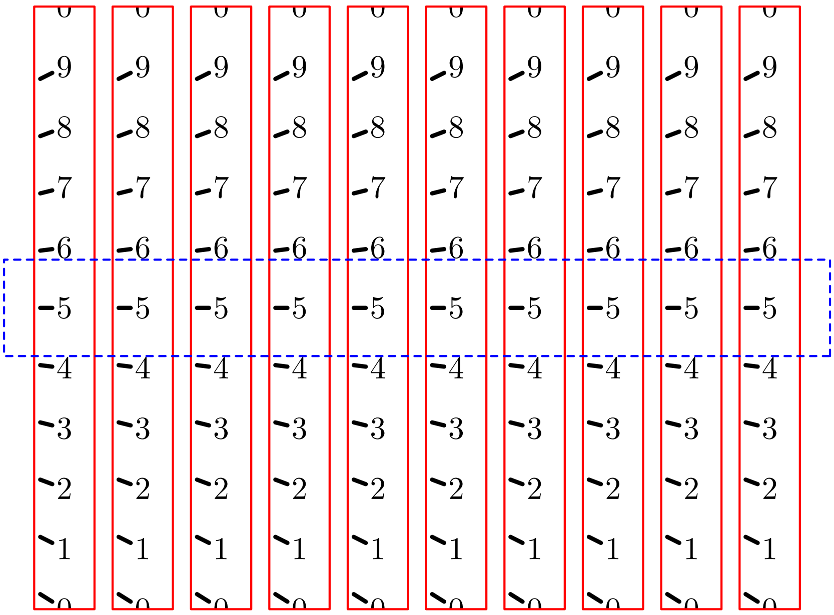


Figure 15: The representation of 5555555555 in a 5-configuration.

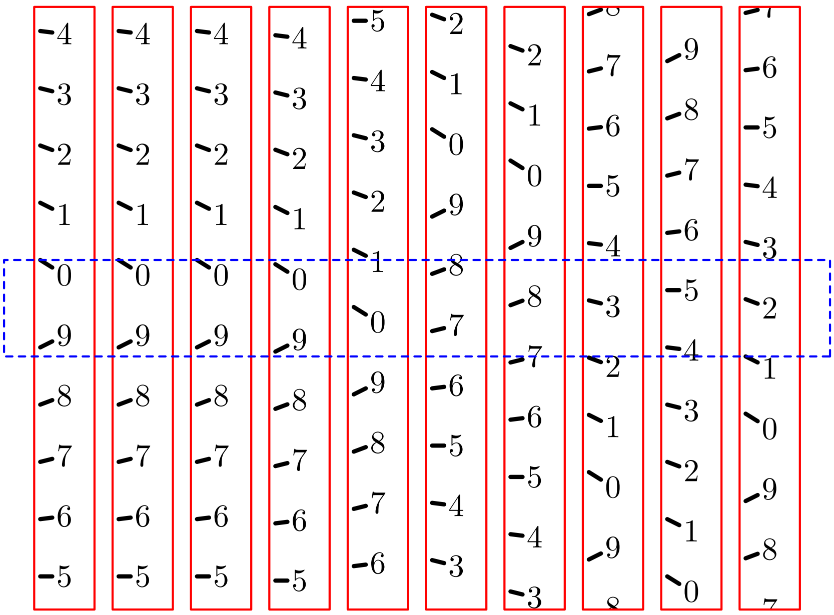


Figure 16: The representation of 78352 in a 5-configuration.

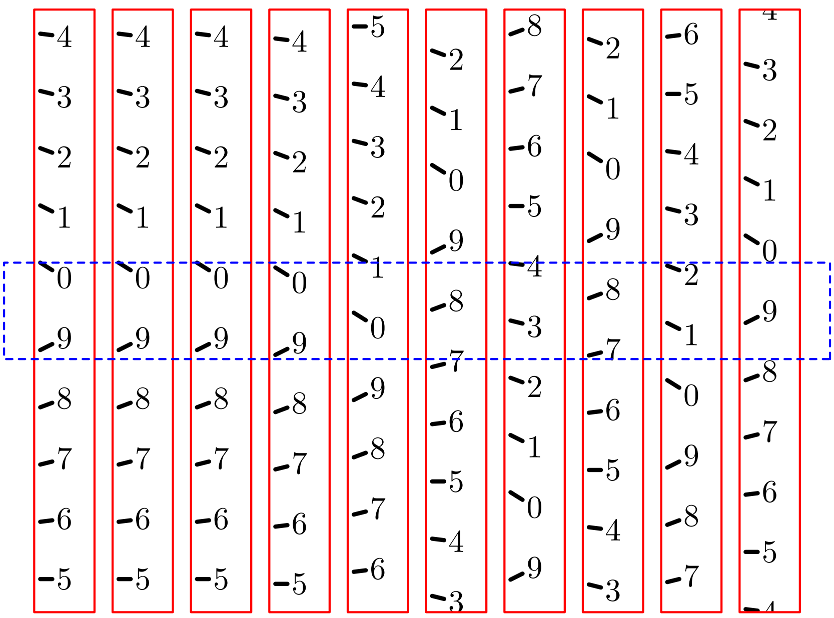


Figure 17: Adding 5467 to 78352 in a 5-configuration, giving 83819.

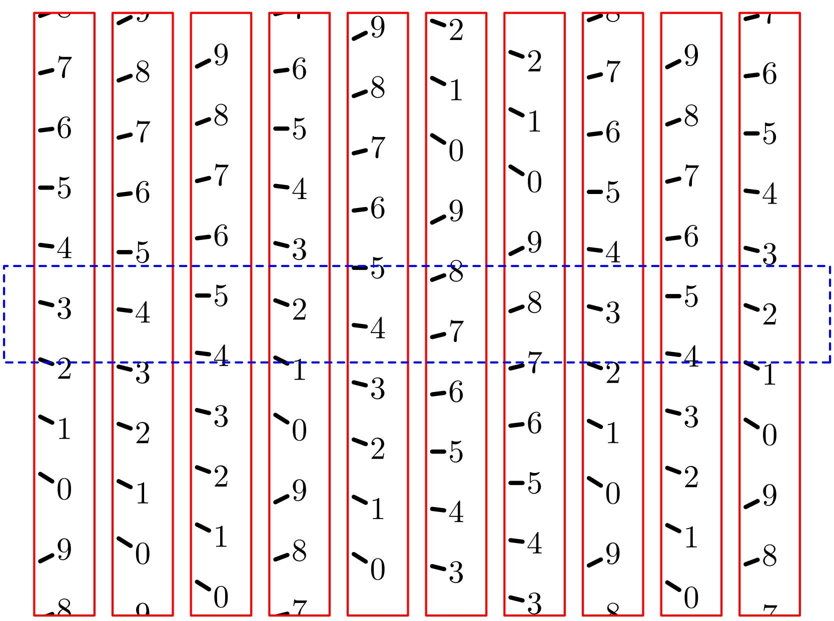


Figure 18: The representation of 3452478352 in a 5-configuration.

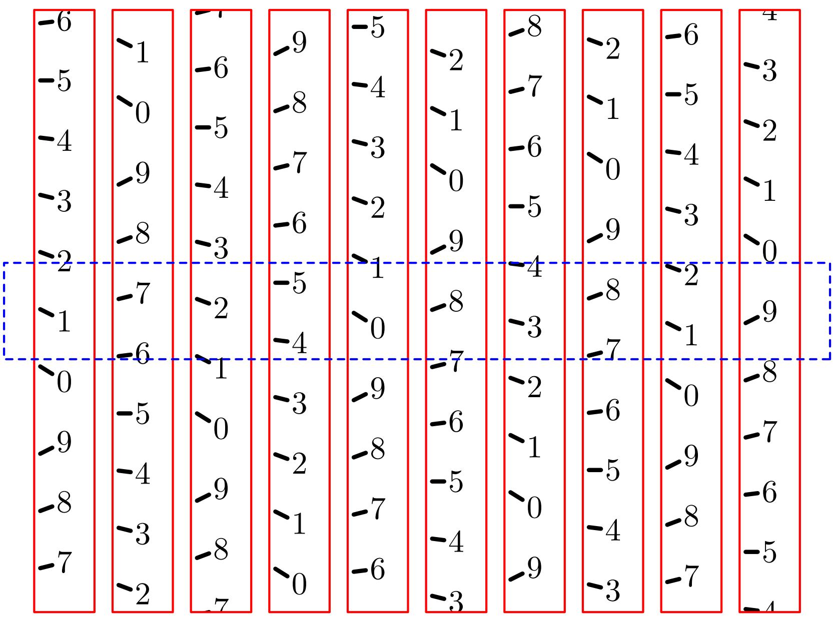


Figure 19: The representation of 1725083819 in a 5-configuration.

6 The effect of play

In order to study the influence of backlash, we can assume that the deviation of one figure wheel with respect to the previous one is no larger than a certain amount. For instance, if the first wheel displays 5, the second wheel should normally display an integer, but it may be off by some amount. These deviations are going to accumulate and if there is too much play, the value at the left may be totally wrong. In order to illustrate the problem, consider again the value 1725083819 (figure 19). Now, if we add 0.1 to the first wheel, 0.2 to the second one, 0.3 to the third one, etc., we obtain the configuration of figure 20. The value read is 2836093819 which is not only wrong, but ambiguous to read. For instance, when the digit 6 is read, it is not totally clear if the value to read on its left is 3, or 2, because 3 has deviated too much to the top. This shows that a deviation of 0.1 units is already too much in the worst case. It would however be possible to some extent to read the correct value, if we could read outside of the horizontal window, but that is then far from practical!

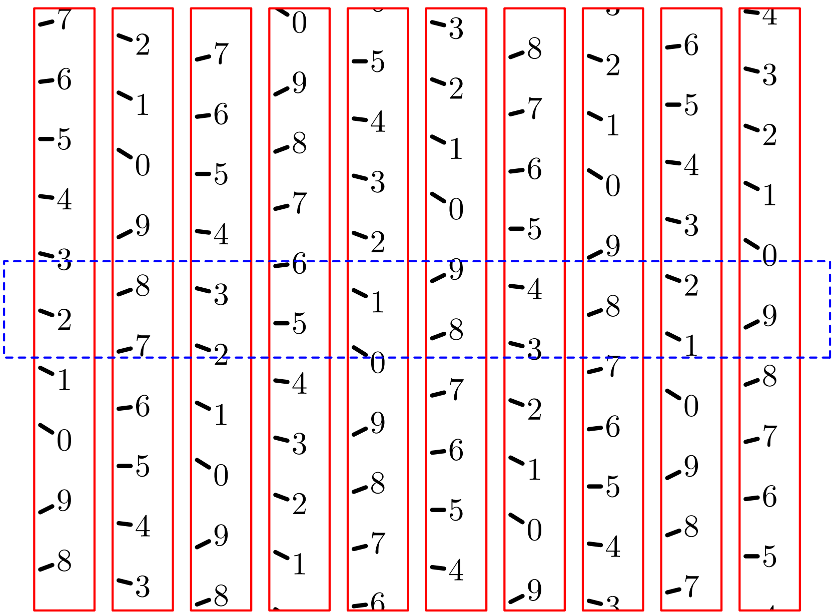


Figure 20: The representation of 1725083819 in a 5-configuration, but with an incremental backlash of 0.1 per digit. The value reads more like 2836093819.

For a good and unambiguous reading, we may assume that the last figure on the left should not be wrong by more than 0.1 or perhaps 0.2

units. In the worst case, this entails that the basic deviation from one wheel to the next should not exceed about 0.02 units. If however we assume that the deviations are more randomly distributed, then a deviation of 0.05 or even 0.1 might be acceptable from one figure wheel to the next one.

These deviations are essentially due to the four wheels a, b, c and d, because the carrier wheels are kept in place by springs and their deviations cannot accumulate [15, 18].

7 Setting the machine to 0

The adding machine can easily be set to the value representing 0, by setting each wheel appropriately from right to left. This is made easy by notches on each figure wheel, and claws which can be activated to enter these notches [15]. When turning the first figure wheel backwards, we reach 0, and then a claw enters the notch, blocking the wheel. The second figure wheel is then turned (using the receiving wheel), and it too will reach its initial position and be stopped. And so on. The claws are then removed and the machine can again be operated. This is easier than trying to set the machine on the value representing 0, for the very reason that 0 is not represented by a mere alignment of zeroes.

8 Chebyshev's extension for multiplication

Chebyshev's second adding machine (figures 8 and 9) was actually designed so that it could become a component of a larger machine, for multiplying or dividing (figures 21 and 22). This machine was built in 1882. It is quite interesting, and perhaps unique in the history of computing, to have the possibility of such a separation. The entire machine, with its addition component, is kept in the *Conservatoire des Arts et Métiers* in Paris [6].

In order to use the adding machine as an element of another machine, the carrier wheels are actually toothed, so that pinions can mesh with them. In figure 21, the number to be multiplied is written on the right, and it positions other sliding pinions with respect to a central (Leibniz) cylinder made of 0, 1, 2, ..., and 9 teeth. The multiplicand can contain seven digits and the highest digit is the one given on the first row. Each of the seven figures has a corresponding steel arbor and in addition to the sliding pinions, these seven arbors end with 4 tooth pinions, which are those meshing with seven receiving wheels of the addition module.

The sliding pinions can slide in a slit made in their arbor. Therefore, even though their position may vary, their rotation will still cause their arbor to rotate [15, p. 202].

The core cylinder is rotated by a small crank visible on the right, and it moves accordingly each sliding pinion. If the sliding pinion is positioned on 7, for instance, one turn of the crank will cause another pinion at the end of the same arbor to advance the *total* by $7 \cdot 10^6$.

On another part of the machine, the user can set the multiplier, and it will be used to shift the addition block, or carriage, whenever the necessary turns of the crank have been made. In other words, moving the crank produces at the same time the multiplication by one digit, and moves the addition block.[3]

This machine can also be used for dividing. In that case, the dividend has to be set on the addition carriage before it is put in place in the machine.

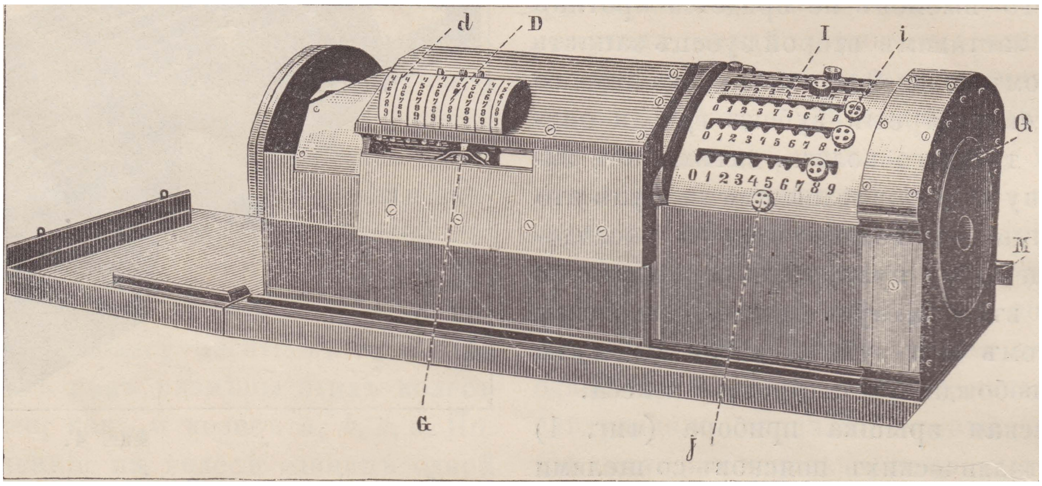


Figure 21: A view of the complete second machine. The addition component can be inserted at the left. (excerpt from [19])

According to Chebyshev, it is not possible to act simultaneously on two consecutive receiving wheels of the adding machine, and the multiplier takes this into account, in that it alternates the motion of the even and odd receiving wheels [15, p. 203]. However, we don't really understand why such simultaneous action is impossible, and it seems to us that simultaneous rotations of two consecutive receiving wheels merely accumulate and

[3]For more details on this very subtle feature, see the description given by D'Ocagne [15, pp. 205–207].

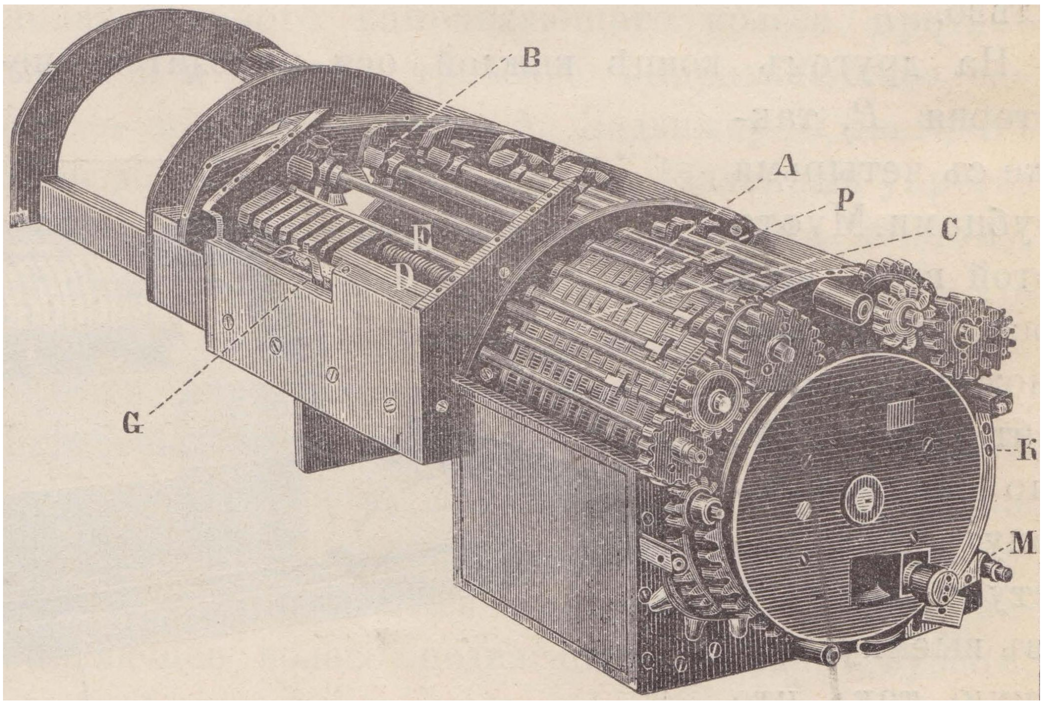


Figure 22: Detail of the complete second machine. The addition component can be inserted at the left. (excerpt from [19])

produce the same final result as non simultaneous rotations. Did we miss something?

9 Recent reconstructions

Thomas Püttmann has recently (2015) built a (simplified) model of Chebyshev's adding machine using the *Fischertechnik* construction set. However, he did not align the figures the way Chebyshev did.

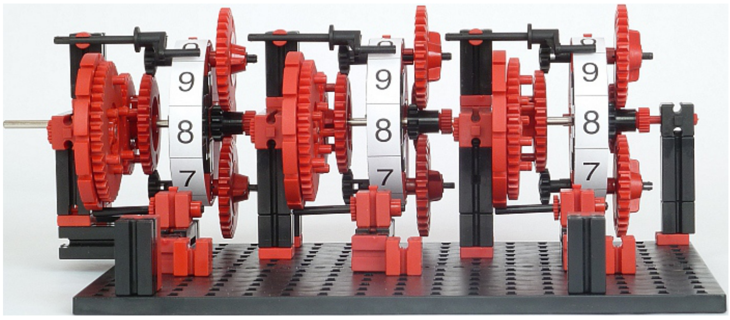


Figure 23: Püttmann's simplified reconstruction of Chebyshev's adding machine. (`https://fischertechnikblog.wordpress.com`)

10 Conclusion

Chebyshev's adding machine is an unusual calculating machine. Using non intermittent motion, it dispenses with carry levers and such, and allows for faster calculations. It is also a modular machine, in which an independent addition component becomes a sliding carriage of a larger multiplication machine. Chebyshev provided means to read the result of his machine, even though this at first may seem difficult to achieve. Finally, he automated the multiplication by one digit and the shift of the carriage, so that the user merely has to rotate the crank and not worry about shifting when multiplying by numbers greater than 9.

Chebyshev's machine, although very interesting, seems to have been neglected, and it has only very recently regained interest, in particular through the site `http://tcheb.ru`, and a few other studies [9]. Hopefully, the multiplication machine located in Paris will sometime in the future be studied more thoroughly, because it certainly deserves it!

Acknowledgements

This work was inspired by Alain Guyot's description of Chebyshev's machine, and Georgi Dalakov kindly provided a copy of Владимир Георгиевич фон Бооль's 1896 description of the machine.

References

[1] Иван Иванович Артоболевский (I. I. Artobolevsky) and Николай Иванович Левитский (N. I. Levitsky). Механизмы П. Л. Чебышева (= Chebyshev's mechanisms). In Научное наследие П. Л. Чебышева *(= P. L. Chebyshev's Scientific Heritage)*, volume 2, page 236. Москва-Ленинград: Изд-во АН СССР, 1945.

[2] Иван Иванович Артоболевский (I. I. Artobolevsky) and Николай Иванович Левитский (N. I. Levitsky). Модели механизмов П. Л. Чебышева (= Models of mechanisms of P. L. Chebyshev). In Полное собрание сочинений П. Л. Чебышева *(= The complete works of P. L. Chebyshev)*, volume 4, page 242. Москва-Ленинград: Изд-во АН СССР, 1948.

[3] Paul L. Butzer and François Jongmans. P. L. Chebyshev (1821–1894) and his contacts with Western European scientists. *Historia Mathematica*, 16:46–68, 1989.

[4] Paul L. Butzer and François Jongmans. P. L. Chebyshev (1821–1894): A guide to his life and work. *Journal of Approximation Theory*, 96:111–138, 1999.

[5] Пафнутий Львович Чебышёв (Chebyshev). Счётная машина с непрерывным движением (= Calculating machine with continuous movement). In Полное собрание сочинений П. Л. Чебышева *(= The complete works of P. L. Chebyshev)*, volume 4, pages 158–160. Москва-Ленинград: Изд-во АН СССР, 1948.

[6] Conservatoire national des arts et métiers. *Catalogue du musée, section A: Instruments et machines à calculer*. Paris: Conservatoire national des arts et métiers, 1942. [Chebyshev's machine is described on pages 67–69, with figures 36–38].

[7] Anne-Marie Décaillot. L'arithméticien Édouard Lucas (1842–1891) : théorie et instrumentation. *Revue d'histoire des mathématiques*, 4:191–236, 1998.

[8] Anne-Marie Décaillot-Laulagnet. *Édouard Lucas (1842–1891) : le parcours original d'un scientifique français dans la deuxième moitié du XIX^e siècle*. PhD thesis, Université René Descartes-Paris V, 1999.

[9] Alain Guyot. La machine à calculer continue de Tchebychev et le reporteur différentiel, 2014.

http://www.aconit.org/histoire/calcul_mecanique/documents/Tchebichef_1882.pdf.

[10] Eugène Lemaire. Exposition publique de machines à calculer anciennes et modernes. *Bulletin de la Société d'encouragement pour l'industrie nationale*, 132:608–644, septembre-octobre 1920. [pp. 616–617 on Chebyshev's machines].

[11] Édouard Lucas. *Récréations mathématiques*, volume 3. Paris: Gauthier-Villars et fils, 1893. [p. 74 briefly describes Chebyshev's machine].

[12] Леонид Ефимович Майстров (Leonid Maistrow) and Ольга Леонидовна Петренко (Olga Petrenko). Приборы и инструменты исторического значения: вычислительные машины *(= Apparatus and instruments of historical importance: computing machines)*. Москва: Машиностроение, 1981. [not seen].

[13] Jean Marguin. *Histoire des instruments et machines à calculer : Trois siècles de mécanique pensante, 1642–1942*. Paris: Hermann, 1994.

[14] Ernst Martin. *Die Rechenmaschinen und ihre Entwicklungsgeschichte*. Pappenheim: Meyer, 1925. [reprinted in 1986, and English translation in 1992 under the title "The calculating machines (Die Rechenmaschinen): their history and development", MIT Press] [see pp. 88–91 on Chebyshev's machine].

[15] Maurice d'Ocagne. *Le calcul simplifié par les procédés mécaniques et graphiques*. Paris: Gauthier-Villars, 1905. [second edition; first edition in 1894, third edition in 1928, and English translation in 1986].

[16] Maurice d'Ocagne. *Vue d'ensemble sur les machines à calculer*. Paris: Gauthier-Villars et Cie, 1922.

[17] Maurice d'Ocagne. *Hommes & choses de science : propos familiers*. Paris: Vuibert, 1930. [pp. 26–27 briefly mention Chebyshev's machine].

[18] P. Tchébychef. Une machine arithmétique à mouvement continu. *Revue scientifique de la France et de l'Étranger*, 30(13):402–404, 1882.

[19] Владимир Георгиевич фон Бооль (V. G. von Bool). Арифмометр Чебышева (= Chebyshev's arithmometer). Известия Императорского общества любителей естествознания, антропологии и этнографии, состоящего при Императорском

Московском университете *(= Proceedings of the Imperial Society of naturalists, anthropologists, and ethnographers, affiliated with the Imperial Moscow University)*, 91(1):12–22, 1894. [Труды Отделения физических наук общества любителей естествознания (= Proceedings of the physics department of the society of devotees of natural sciences), volume 7, issue 1].

[20] Владимир Георгиевич фон Бооль (V. G. von Bool). Арифмометр Чебышева (= Chebyshev's arithmometer). Вестник опытной физики и элементарной математики *(= Bulletin of experimental physics and elementary mathematics)*, 18(205):5–10, 1895.

[21] Владимир Георгиевич фон Бооль (V. G. von Bool). Арифмометр Чебышева (= Chebyshev's arithmometer). Вестник опытной физики и элементарной математики *(= Bulletin of experimental physics and elementary mathematics)*, 18(207):52–56, 1895.

[22] Владимир Георгиевич фон Бооль (V. G. von Bool). Арифмометр Чебышева (= Chebyshev's arithmometer). Вестник опытной физики и элементарной математики *(= Bulletin of experimental physics and elementary mathematics)*, 18(210):134–141, 1895.

[23] Владимир Георгиевич фон Бооль (V. G. von Bool). Приборы и машины для механическаго производства арифметических действий. Описание и оценка счетных приборов и машин *(= Devices and machines for the production of mechanical arithmetic. Description and evaluation of computing devices and machines)*. 1896. [pp. 110–133 on Chebyshev's machine].

[24] Alexander Vucinich. *Science in Russian culture, 1861–1917*. Stanford: Stanford University Press, 1970. [p. 167 briefly mentions Chebyshev's machine].